Instability of Enclosed Horizons

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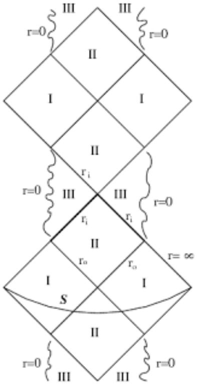
Reissner-Nordström Cauchy Horizon Instability

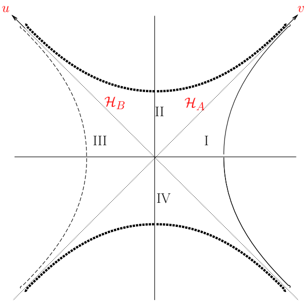
(Simpson-Penrose 1973, Hawking-Ellis 1973, Chandrasekhar-Hartle 1982)
For (say) wave equation on RN, compactly supported initial perturbations lead to singularity in T_{ab} at the Cauchy horizon. (Also for natural Hadamard quantum states, the quantum T_{ab} diverges there.)

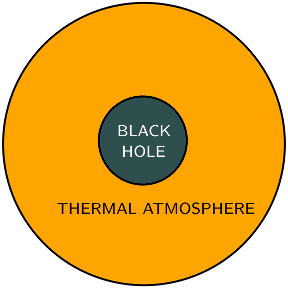


This suggests that in full GR, the Cauchy horizon is unstable. In fact (Poisson-Israel 1990, Dafermos 2001+) generically, it seems the metric is C^0 but not C^1 extendable. Expectation (?): The classical spacetime description breaks down below the Cauchy horizon and the region beyond the Cauchy horizon is an unphysical fiction.

- Simple indication of all this: An observer crossing the Cauchy horizon will see an infinite amount of history in a finite amount of time (just before crossing the horizon).

Now consider the wave equation on the region of Kruskal to the left of a constant Schwarzschild-radius sphere in the right wedge with, say, vanishing boundary conditions on that sphere [1].

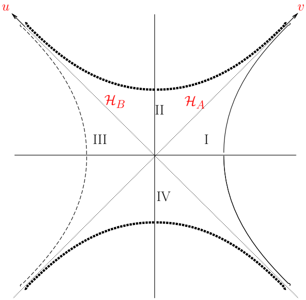




- An observer crossing the right past event horizon will see an infinite amount of history in a finite amount of time (now just *after* crossing the horizon).

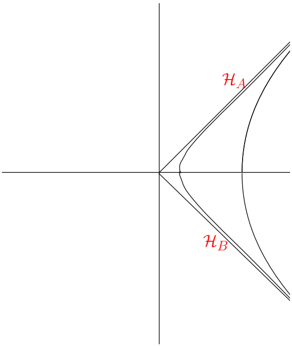
[1] Sometimes we will also consider an image box in the left wedge – see dotted line.

By analogy with RN, this suggests that, for suitable initial data, there may be a similar singularity in T_{ab} – now on the right past event horizon – suggesting that this spacetime



is unphysical.

... and should be replaced by this spacetime (with classical describability breaking down in a region as indicated near the horizon).

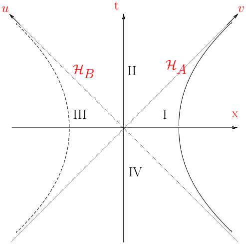


A 1+1 dimensional model

- We obtain results for the wave equation

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = 0$$

  on the region of 1+1 Minkowski to the left of an eternally accelerated mirror (or between two such mirrors) – with Dirichlet boundary conditions on the mirror(s).
- They turn out to be weaker than the RN results. But still arguably strong enough to support our above suggestions.



- We expect similar results to hold for asymptotically null enclosures when have compact "undepicted" dimensions. This includes Schwarzschild in a box and also Schwarzschild-AdS – whose conformal boundary provides a natural box.

Result 1: Let $u = t - x$, $v = t + x$.
Wave equation becomes

$$\frac{\partial^2 \phi}{\partial u \partial v} = 0$$

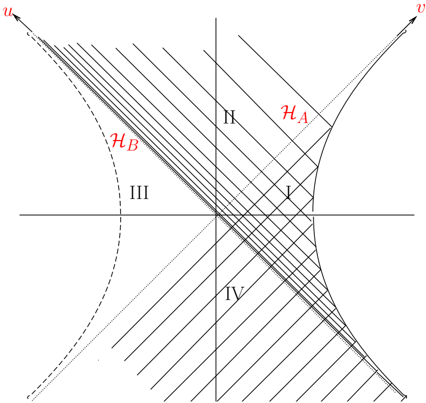
with general solution

$$\phi(u, v) = f(u) + g(v)$$

where, by the boundary conditions ($uv = -1$),

$$g(v) = -f(-1/v).$$

In picture, (real parts understood) $f(u) = e^{-i\omega u}$, $g(v) = -e^{i\omega/v}$ ($u > 0$ suppressed).



Pile-up leads to $T_{vv} = \left(\frac{\partial \phi}{\partial v}\right)^2$ singular on $\mathcal{H}_B$ (i.e. at $v = 0$).

Of course, the incoming wave here doesn't have finite energy, but you can modify it to have finite energy:

Result 2 : Take $f_p(u) = C(u^2 + a^2)^{-p/2}e^{-i\omega u}$ for $p > 1/2$. You can check that, for $p < 2$, we still have T_{vv} singular at $v = 0$. (Also, btw, for $p > 3/2$ the total integrated energy of the reflected wave will be finite.)

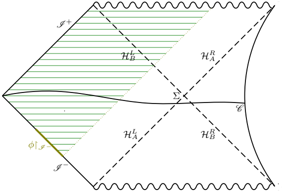
Result 3: For compactly supported initial data, as small as you like, on a suitable early (almost) Cauchy surface, T_{ab} can be as large as you like (but not infinite) near the horizon. Details on blackboard.

Result 4: For the quantum theory in the case of two mirrors, there is a stationary Hadamard Hartle-Hawking-Israel-like "vacuum" state, ω with $\omega(T_{ab}))^{\mathrm{ren}}$ finite (in fact it is zero!). However, the coherent state $W(\phi_p)\Omega$ $(= e^{-i\sigma(\hat{\phi},\phi_p)}\Omega)$ built on the GNS vacuum vector, Ω, will have $\langle T_{vv}\rangle^{\mathrm{ren}} = T_{vv}^{\mathrm{classical}}$ and thus have the same singularity.

Result 5: (Joint work with Umberto Lupo.) For the quantum theory in the case of one mirror, there is no stationary Hadamard state (for suitable notion of Hadamard). And we conjecture that there is no Schwarzschild-isometry-invariant Hadamard state on region of Kruskal to left of box. (Again for suitable notion of Hadamard in presence of boundaries.)

For the full argument I refer to the paper. Suffice it to mention here that it relies on:

(A) (Up to technicalities about 'tails') because of reflection at the box, the set of classical solutions in Region I which fall entirely through the right A-horizon $\mathcal{H}_A^R$ will coincide with the set of classical solutions which fall entirely through the right B-horizon $\mathcal{H}_B^R$.



(B) There exists a classical solution, ϕ^L supported in Regions II and III, which vanishes on $\mathcal{H}_A$ but not on $\mathcal{H}_B^L$

Comparison between Reissner-Nordström results and Results 1-4 for enclosed horizons

Reissner-Nordström Cauchy Horizon: The small initial perturbations for which T_{ab} is singular on the Cauchy horizon include those with compactly supported Cauchy data.

Also (Hiscock 1977) the natural quantum vacuum state has singular $\langle T_{ab} \rangle_{\mathrm{ren}}$ at the Cauchy horizon. (See also results by Benito Juárez-Aubry.)

Minkowski enclosed by accelerated mirror(s): The singularity in T_{ab} at the horizon requires a finite-energy tail. For compactly supported initial data, only get "almost singularity".

Natural HHI state non-singular, albeit coherent states built on it are singular/almost singular.

Physical Significance: All the above suggests . . .

When we switch on Newton's constant, the classical theory breaks down at the horizons. The right wedge of enclosed Kruskal become a spacetime in its own right with a non-classically describable region near where the horizons used to be[2].

It seems reasonable to then assume that, for enclosed Kruskal or Schwarzschild-AdS, the (matter part of the) thermal atmosphere in the right wedge is no longer entangled with the matter in the left wedge. Instead, if the overall state is to be pure, it must be entangled with something and that something can only be the gravitational field on the right wedge!

This would be in accordance with my 1998 matter-gravity entanglement hypothesis according to which the state of quantum gravity for an enclosed black hole is a pure quantum state of quantum gravity, entangled between matter and gravity in such a way that the reduced states of the matter and of the gravity are each approximately thermal. (In contrast to the usual belief that the total state is a thermal state.)

[2] A similar picture was arrived at by string-theorists Mathur and Avery and Chowdhury for Schwarzschild-AdS in 2014/2013.